VanillaICE: Hidden Markov Models for the Assessment of Chromosomal Alterations using High-throughput SNP Arrays

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Abstract

This package provides an implementation of a hidden Markov Model to identify copy number alterations from high throughput SNP arrays.

1 From processed SNP summaries

The starting point for this section of the vignette are B allele frequencies and log R ratios that are available from software such as GenomeStudio and the R package *crlmm*. In this section, we assume that the low-level summaries are available in a plain text file – one file per sample. For users of the *crlmm* package for preprocessing, please refer to the *crlmmDownstream* vignette.

To illustrate the workflow for copy number analysis, this package provides Illumina 610k array data for one Hapmap trio and one oral cleft trio preprocessed by GenomeStudio [1]. To keep the size of this package small, the GenomeStudio-processed files for these 6 samples have been parsed to include only columns relevant to copy number analyses and $\approx 11 \text{k}$ markers that are spanned by or border putative CNVs. The code chunks provided below would be the same if the full files were included.

```
> library(VanillaICE)
> library(foreach)
> registerDoSEQ()
> extdir <- system.file("extdata", package="VanillaICE", mustWork=TRUE)
> files <- list.files(extdir, pattern="FinalReport")</pre>
```

The first several lines of these files have the following information:

```
[Header]
BSGT Version,3.3.7
Processing Date,5/2/2009 12:08 AM
Content,,Human610-Quadv1_B.bpm
Num SNPs,620901
Total SNPs,620901
Num Samples,7599
Total Samples,7599
File ,1664 of 7599
[Data]
SNP Name,Allele1 - AB,Allele2 - AB,B Allele Freq,Log R Ratio cnvi0005126,-,-,0.0085,-0.7214
cnvi0005127,-,-,0.0081,-0.8589
cnvi0005128,-,-,0.0063,-0.164
```

The physical annotation of the markers in the genome can be downloaded from on-line resources or from annotation packages available from Bioconductor. Here, we provide a plain-text file containing the genomic annotation for the markers based on build UCSC build hg18:

```
> list.files(extdir, pattern="SNP_info")
[1] "SNP_info.csv"
```

The annotation file contains the the name of the SNP or nonpolymorphic marker ('Name'), an indicator for whether the marker is a SNP ('Intensity Only'), the chromosome name ('Chr'), and the genomic physical position ('Position'). The columns for the SNP annotation can be in any order and have different variable labels than the file provided with this package. Below, we copy the header of the provided annotation file:

```
Chr, Position, Name, Intensity Only 11,45380778, cnvi0005126,1 11,45382079, cnvi0005127,1 11,46561032, cnvi0005128,1
```

1.1 Organizing marker annotation

We require that marker-level annotation is represented as a GRanges-derived class. To read the plain text annotation file, we use the fread provided in the data.table package. In addition, we define an indicator for whether the marker is polymorphic using the 'Intensity Only' flag. The SnpGRanges class created from the fgr object in the following code-chunk ensures that this binary indicator is created and can be reliably accessed in downstream processing.

Ideally, one should also include the genome build and information on the chromosome lengths appropriate to the build. Here, we extract the metadata on the chromosomes using BSgenome Bioconductor package for the hg18 build. Finally, we sort the fgr object such that the chromosomes are ordered by their seqlevels and the markers are ordered by their genomic position along the chromosome.

```
> library(BSgenome.Hsapiens.UCSC.hg18)
> sl <- seqlevels(BSgenome.Hsapiens.UCSC.hg18)
> seqlevels(fgr) <- sl[sl %in% seqlevels(fgr)]
> seqinfo(fgr) <- seqinfo(BSgenome.Hsapiens.UCSC.hg18)[seqlevels(fgr),]
> fgr <- sort(fgr)</pre>
```

1.2 Organizing the marker-level summaries

The abbreviated plain text files included with this package and containing log R ratios and B allele frequencies from 2 trios are listed below.

```
> files <- list.files(extdir, full.names=TRUE, recursive=TRUE, pattern="FinalReport")
```

We will parse these files into a specific format such that all downstream steps for copy number estimation no longer depend on the specific format of the source files. At this point, we can encapsulate the the names of the source files ('sourcePaths'), the location of where we intend to store the parsed data ('parsedPath'), and the genomic marker annnotation created in the preceding section in a single object called an ArrayViews.

```
> ##
> ## A directory to keep parsed files
> ##
> parsedDir <- tempdir()
> views <- ArrayViews(rowRanges=fgr, sourcePaths=files, parsedPath=parsedDir)
> lrrFile(views) <- file.path(parsedDir, basename(fileName(views, "lrr")))
> views@bafFiles <- file.path(parsedDir, basename(fileName(views, "baf")))
> views@gtFiles <- file.path(parsedDir, basename(fileName(views, "gt")))
> colnames(views) <- gsub(".csv", "", colnames(views))
> show(views)

class 'ArrayViews'
   No. files : 6
   No. markers: 11780
```

Because the format of the source files depends on upstream software and version number within software, our approach to parsing these files is to read one file and to store the appropriate metadata from this file so that subsequent files can be parsed in a similar fashion. We use the fread to read in the first file.

```
> ## read the first file
> dat <- fread(files[1])
> head(dat,n=3)

SNP Name Allele1 - AB Allele2 - AB B Allele Freq Log R Ratio
1: cnvi0005126 - 0.0085 -0.7214
2: cnvi0005127 - 0.0081 -0.8589
3: cnvi0005128 - 0.0063 -0.1640
```

Next, we select which columns we plan to keep. Again, the required data for downstream processing is the name of the SNP identifier, the log R ratios, and B allele frequencies.

We also specify the order in which we will store the marker-level summaries by matching the row-names of the views object with the names of the markers in the source file:

```
> index_genome <- match(names(fgr), dat[["SNP Name"]])</pre>
```

Similar to the parameter classes defined in *Rsamtools*, we encapsulate the information for parsing the columns and rows of the source files in a class. In addition, we specify which variable names in the source file refers to log R ratios ('cnvar'), B allele frequences ('bafvar'), and genotypes ('gtvar').

The parseSourceFile will parse a single file in the views object (by default, the first file) according to the parameters for reading the data in the scan_params object and write the output to the parsedPath directory. In particular, the parseSourceFile returns NULL.

```
> parseSourceFile(views, scan_params)
```

Apart from confirming their existance, the user should not have a need to directly access the parsed files. Utilities for querying these files are provided through the views object.

```
> head(list.files(parsedPath(views)), n=3)
```

- [1] "FinalReport1664_baf.rds" "FinalReport1664_gt.rds"
- [3] "FinalReport1664_lrr.rds"

1.3 Accessors for the parsed data

The preference for writing the parsed data to disk rather than keeping the data in RAM is simply that the latter does not scale to projects involving thousands of samples. For the former, slices of the parsed data easily be accessed from the parsedPath directory via methods defined for the ArrayViews class. For example, one can use accessors for the low-level summaries directly: lrr, baf, and genotypes for log R ratios, B allele frequencies, and genotypes, respectively. The user has the option of either subsetting the views object or subsetting the matrix returned by the accessor to extract the appropriate data slice. In the following examples, we access data on the first 2 markers.

> lrr(views)[1:2,]

```
FinalReport1664.txt FinalReport1675.txt
rs12789205
                          0.270
                                              -0.168
rs2114088
                          0.114
                                              -0.182
           FinalReport1686.txt FinalReport6841.txt
rs12789205
                          0.190
                                              -0.083
                         -0.114
rs2114088
                                               0.252
           FinalReport6872.txt FinalReport6903.txt
rs12789205
                          0.067
                                              -0.101
rs2114088
                         -0.076
                                              -0.042
> ## or
> lrr(views[1:2, ])
           FinalReport1664.txt FinalReport1675.txt
rs12789205
                          0.270
                                              -0.168
rs2114088
                          0.114
                                              -0.182
           FinalReport1686.txt FinalReport6841.txt
rs12789205
                          0.190
                                              -0.083
rs2114088
                         -0.114
                                               0.252
           FinalReport6872.txt FinalReport6903.txt
rs12789205
                          0.067
                                              -0.101
rs2114088
                         -0.076
                                              -0.042
> ## B allele frequencies
> baf(views[1:2, ])
```

```
FinalReport1664.txt FinalReport1675.txt
                          0.998
rs12789205
                                               0.528
rs2114088
                          0.000
                                               0.000
           FinalReport1686.txt FinalReport6841.txt
rs12789205
                          0.548
                                               0.48
rs2114088
                          0.000
                                               0.00
           FinalReport6872.txt FinalReport6903.txt
rs12789205
                          0.481
                                               0.487
                          0.000
                                               0.492
rs2114088
> ## Use :: to avoid masking by function of the same name in crlmm
> VanillaICE::genotypes(views)[1:2, ]
           FinalReport1664.txt FinalReport1675.txt
rs12789205
rs2114088
                              1
           FinalReport1686.txt FinalReport6841.txt
rs12789205
rs2114088
           FinalReport6872.txt FinalReport6903.txt
rs12789205
                              2
                                                   2
rs2114088
                              1
                                                   2
```

More often, it is useful to extract the low-level data in a SummarizedExperiment-derived class such that meta-data on the samples remains bound to the columns of the assay data (log R ratios / B allele frequencies) and meta-data on the rows remains bound to the rows of the assay data. This is accomplished by applying the SnpExperiment function to a views object. In the following example, we create a SummarizedExperiment-derived class for the first three samples in the views object.

```
> snp_exp <- SnpExperiment(views[, 4:5])
> show(snp_exp)

class: SnpArrayExperiment
dim: 11780 2
exptData(0):
assays(2): cn baf
rownames(11780): rs12789205 rs2114088 ... rs5994329 rs1055232
rowRanges metadata column names(1): isSnp
colnames(2): FinalReport6841.txt FinalReport6872.txt
colData names(0):
```

2 Hidden Markov model

2.1 HMM parameters

The hidden Markov model for inference of germline copy number from SNP array data was originally described in Scharpf *et al.* [3] but has evolved to focus more on B allele frequencies than genotype call probabilities [the latter requires a dependency on the *crlmm* package]. Many of the the updates for computing emission probabilities have been described more recently [2].

Fitting the hidden Markov model to the marker-level summaries requires extracting the relevant data from a views object using the SnpExperiment function illustrated in the previous section. All user-level parameters relevant for fitting the HMM are specified in a parameter object for the emission

probabilities and a parameter object for the transition probabilities. In the following code-chunk, we create a parameter object for the emission probabilities.

```
> param <- EmissionParam()</pre>
```

The easiest way to specify or change parameters for computing emission probabilities is through the EmissionParam constructor. For example, here we create a parameter object with the temper parameter set to 1/2:

```
> param <- EmissionParam(temper=0.5)
> show(param)

EmissionParam :
CN mean: -2, -0.4, 0, 0, 0.4, 1
CN sd: 0.6, 0.3, 0.3, 0.3, 0.3, 0.3
BAF mean: 0, 0.25, 0.33, 0.5, 0.67, 0.75, 1
BAF sd: 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1
max # of EM updates: 5
initial state probabilities: 0.17, 0.17, 0.17, 0.17, 0.17
tempering scalar: 0.5
model homozygous regions: FALSE
See temper(), cn_means(), cn_sds(),...
```

Similarly, the constructor for the TransitionParam class can be used to specify alternatives to the defaults. By not specifying an TransitionParam object, default values created by the constructor are used by the HMM in the following example.

2.2 Fitting the HMM

The function hmm2 fits the 6-state HMM to the snp_exp object, returning a list-derived class (a HMMList) of the same length as the number of samples.

```
> fit <- hmm2(snp_exp, param)</pre>
> show(fit)
An object of class 'HMMList'
 Length: 2
 Use [[j]] to retrieve results for the jth sample.
 Use unlist to coerce to a 'GRanges' object.
> length(fit)
[1] 2
> ## HMM object for the first sample
> fit[[1]]
Object of class 'HMM'
 granges (no. segments) : 7
 no. duplications
 no. hemizygous deletions: 0
 no. homozygous deletions: 0
See posterior(), segs(), cnvSegs(), getHmmParams(),
```

3 Inspecting, Filtering, and plotting HMM results

Several methods are defined for the HMMList class to facilitate the selection of candidate CNVs, the filtering of technical artifacts, and to enable visualization of the HMM calls in the context of the marker-level summaries.

The fit object is an instance of the HMMList object of length three, each element corresponding to a sample:

```
> names(fit)
```

[1] "FinalReport6841.txt" "FinalReport6872.txt"

The R method \\ can be used to access the HMM summary for the results for the jth sample. For example, here we extract the HMM summary for the 2nd sample:

```
> show(fit[[2]])
```

```
Object of class 'HMM'
  granges (no. segments) : 13
  no. duplications : 1
  no. hemizygous deletions: 4
  no. homozygous deletions: 0
See posterior(), segs(), cnvSegs(), getHmmParams(),
```

Alternatively, we can use unlist to create a single GRanges-derived class containing the segmentation of all three samples

> head(unlist(fit), n=3)

HmmGRanges object with 3 ranges and 4 metadata columns:

	seqnames		ranges	strand		${\tt numberFeatures}$	state
	<rle></rle>		<pre><iranges></iranges></pre>	<rle></rle>		<integer></integer>	<integer></integer>
[1]	chr11	[43132366,	55116789]	*	-	1445	3
[2]	chr11	[55124465,	55204003]	*	-	29	5
[3]	chr11	[55209499,	67164817]	*	-	1963	3
	prCall	L	id				
	<numeric> <character></character></numeric>						
[1]	[1] 1 FinalReport6841.txt						
[2]	1						
[3]	1	l FinalRepor	rt6841.txt				
		_	1 40				

seqinfo: 3 sequences from hg18 genome

```
or a GRangesList by splitting on the sample id:
```

```
> grl <- split(unlist(fit), unlist(fit)$id)</pre>
```

3.1 Filters

There are several meta-data columns stored in the GRanges-derived summary useful for filtering the set of genomic intervals. All the available parameters for filtering the fit object are stored in the parameter class FilterParam of which an instance can be created by its constructor of the same name without any arguments.

```
> filter_param <- FilterParam()
> show(filter_param)

An object of class 'FilterParam'
  min. posterior probability of CNV call: 0.99
  min. no. of markers spanned by segment: 10
  min. width of segment : 1
  selected HMM states : 1, 2, 3, 4, 5, 6
  selected seqnames : chr1, chr2, chr3, chr4, chr5, chr6 ...
```

To apply the default filter parameters to the fit object, we use the cnvFilter function. The cnvFilter function returns only the set of genomic ranges satisfying the filters. For example,

> cnvFilter(fit, filter_param)

HmmGRanges object with 18 ranges and 4 metadata columns:

	seqnames		ranges	strand		${\tt numberFeatures}$
	<rle></rle>		<pre><iranges></iranges></pre>	<rle></rle>		<integer></integer>
[1]	chr11	[43132366,	55116789]	*		1445
[2]	chr11	[55124465,	55204003]	*		29
[3]	chr11	[55209499,	67164817]	*		1963
[4]	chr12	[21054,	17482570]	*		4591
[5]	chr22	[14456412,	23994408]	*		2244
[14]	chr22	[18710895,	19048116]	*		28
[15]	chr22	[19066315,	19819918]	*	I	211
[16]	chr22	[19842333,	23983992]	*	I	1099
[17]	chr22	[23991725,	24236803]	*		63
[18]	chr22	[24237905,	29381906]	*		1441
	state	e prCall		id		
	<integer></integer>	<pre><numeric></numeric></pre>	<	character>		
[1]	3	1	FinalRepor	rt6841.txt		
[2]	5	1	FinalRepor	rt6841.txt		
[3]	3	1	FinalRepor	rt6841.txt		
[4]	3	3 1	FinalRepor	rt6841.txt		
[5]	3	1	FinalRepor	rt6841.txt		
[14]	3	1	FinalRepor	rt6872.txt		
[15]	2	2 1	FinalRepor	rt6872.txt		
[16]	3	1	FinalRepor	rt6872.txt		
[17]	5	1	FinalRepor	rt6872.txt		
[18]	3	3 1	FinalRepor	rt6872.txt		

seqinfo: 3 sequences from hg18 genome

To select only the segments with altered copy number states (states 1, 2, 5, and 6) on chromosome 22, we can define the filter parameters as follows:

```
> select_cnv <- FilterParam(state=c("1", "2", "5", "6"), seqnames="chr22")
> cnvs <- cnvFilter(fit, select_cnv)
> cnvs
```

HmmGRanges object with 6 ranges and 4 metadata columns:

S	eqnames		ranges	strand	ı	numberFeatures	state
	<rle></rle>		<pre><iranges></iranges></pre>	<rle></rle>	-	<integer></integer>	<integer></integer>
[1]	chr22	[23999142,	24240667]	*		65	5
[2]	chr22	[17202486,	17388108]	*		44	2
[3]	chr22	[17405381,	17633332]	*		48	2
[4]	chr22	[17690812,	18693299]	*	-	281	2
[5]	chr22	[19066315,	19819918]	*	-	211	2
[6]	chr22	[23991725,	24236803]	*	-	63	5
prCall id							
<1	<pre><numeric> <character></character></numeric></pre>						
[1]	[1] 1 FinalReport6841.txt						
[2]	[2] 1 FinalReport6872.txt						
[3]	3] 1 FinalReport6872.txt						
[4]] 1 FinalReport6872.txt						
[5]	1 FinalReport6872.txt						
[6]	1	FinalRepor	rt6872.txt				

seqinfo: 3 sequences from hg18 genome

3.2 Visualization

> cnvs_sample2 <- cnvList[[2]]</pre>

> cnvs_sample2

3.2.1 Trellis graphics for low-level summaries

Visualization of the CNVs in the context of the lower-level summaries is acheived through a combination of grid and the grid-derived graphics provided by *lattice*. The xyplotList and constructor HmmTrel-lisParam should be sufficient for producing a decent graphic using the default settings. In the following code chunk, we create a trellis object for each CNV segment.

```
> trellis_param <- HmmTrellisParam()
> cnvList <- split(cnvs, cnvs$id)
> figList <- xyplotList(cnvList, snp_exp, trellis_param)
> names(figList)

[1] "FinalReport6841.txt" "FinalReport6872.txt"

Each element in figList is a trellis object and can be displayed using print.
> class(figList[["FinalReport6841.csv"]][[1]])

[1] "NULL"

3.2.2 Layout using grid
> fig1 <- figList[["FinalReport6841.csv"]][[1]]
> vps <- viewports()
> xygrid(fig1, vps, cnvList[[1]][1])

NULL

A hemizygous deletion on chromosome 22 is split into too many regions.
```

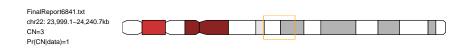
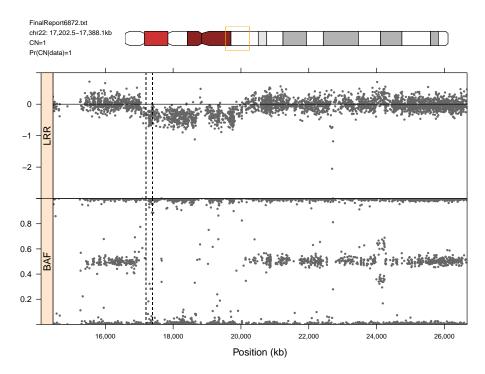


Figure 1: A single-copy duplication (state 5).

HmmGRanges obje	ect with 5	ranges and	4 metad	lata	columns:	
seqnames	ranges	strand	n	umberFeatures	state	
<rle></rle>		<pre><iranges></iranges></pre>	<rle></rle>	1	<integer></integer>	<integer></integer>
[1] chr22	[17202486,	17388108]	*	1	44	2
[2] chr22	[17405381,	17633332]	*	1	48	2
[3] chr22	[17690812,	18693299]	*	1	281	2
[4] chr22	[19066315,	19819918]	*	1	211	2
[5] chr22	[23991725,	24236803]	*	1	63	5
prCall	L	id				
<numeric></numeric>						
[1] 1 FinalReport6872.txt						
[2] 1	[2] 1 FinalReport6872.txt					
[3] 1	[3] 1 FinalReport6872.txt					
[4] 1	[4] 1 FinalReport6872.txt					
[5] 1	l FinalRepo	rt6872.txt				

seqinfo: 3 sequences from hg18 genome

> xygrid(figList[[2]][[1]], vps, cnvs_sample2[1])



Combining the adjacent hemizygous deletions through reduce provides a more satisfying segmentation of this data.

```
> cnvs_sample2r <- reduce(cnvs_sample2, min.gapwidth=500e3)
> fig2 <- xyplotList(cnvs_sample2r, snp_exp)</pre>
```

Parallelization

As the HMM is fit independently to each sample, parallelization is straightforward. In the following unevaluated code chunk, we set up a parallel environment using the R packages <code>snow</code> and <code>foreach</code>.

```
> library(foreach)
> library(snow)
> library(doSNOW)
> c1 <- makeCluster(2, type="SOCK")
> registerDoSNOW(c1)
> results <- hmm2(snp_exp)
> stopCluster(c1)
```

Citing this software

Robert B Scharpf, Giovanni Parmigiani, Jonathan Pevsner, and Ingo Ruczinski. Hidden Markov models for the assessment of chromosomal alterations using high-throughput SNP arrays. *Annals of Applied Statistics*, 2(2):687–713, 2008.

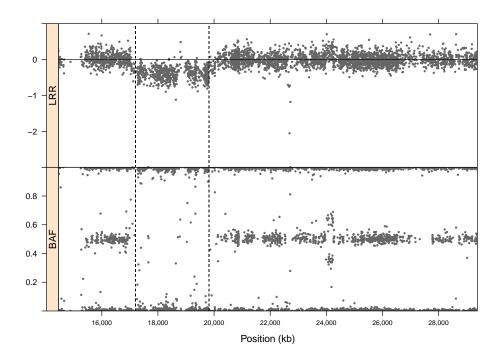


Figure 2: A hemizygous deletion (state 2) on chromosome 22 after merging adjacent hemizygous CNV calls.

Session Information

The version number of R and packages loaded for generating the vignette were:

- R version 3.2.0 alpha (2015-03-20 r68043), x86_64-unknown-linux-gnu
- Locale: LC_CTYPE=en_US.UTF-8, LC_NUMERIC=C, LC_TIME=en_US.UTF-8, LC_COLLATE=C, LC_MONETARY=en_US.UTF-8, LC_MESSAGES=en_US.UTF-8, LC_PAPER=en_US.UTF-8, LC_NAME=C, LC_ADDRESS=C, LC_TELEPHONE=C, LC_MEASUREMENT=en_US.UTF-8, LC_IDENTIFICATION=C
- Base packages: base, datasets, grDevices, graphics, methods, parallel, stats, stats4, utils
- Other packages: BSgenome 1.35.20, BSgenome.Hsapiens.UCSC.hg18 1.3.1000, BiocGenerics 0.13.10, Biostrings 2.35.12, GenomeInfoDb 1.3.16, GenomicRanges 1.19.48, IRanges 2.1.43, S4Vectors 0.5.22, VanillaICE 1.29.12, XVector 0.7.4, data.table 1.9.4, foreach 1.4.2, rtracklayer 1.27.11
- Loaded via a namespace (and not attached): Biobase 2.27.3, BiocInstaller 1.17.7, BiocParallel 1.1.21, GenomicAlignments 1.3.32, Matrix 1.1-5, RCurl 1.95-4.5, Rcpp 0.11.5, RcppEigen 0.3.2.4.0, Rsamtools 1.19.49, VGAM 0.9-7, XML 3.98-1.1, affyio 1.35.0, base64 1.1, bit 1.1-12, bitops 1.0-6, chron 2.3-45, codetools 0.2-11, compiler 3.2.0, crlmm 1.25.0, ellipse 0.3-8, ff 2.2-13, futile.logger 1.4, futile.options 1.0.0, grid 3.2.0, illuminaio 0.9.1, iterators 1.0.7, lambda.r 1.1.7, lattice 0.20-31, matrixStats 0.14.0, mvtnorm 1.0-2, oligoClasses 1.29.6, plyr 1.8.1, preprocessCore 1.29.0, reshape2 1.4.1, splines 3.2.0, stringr 0.6.2, tools 3.2.0, zlibbioc 1.13.3

References

- [1] Terri H Beaty, Jeffrey C Murray, Mary L Marazita, Ronald G Munger, Ingo Ruczinski, Jacqueline B Hetmanski, Kung Yee Liang, Tao Wu, Tanda Murray, M. Daniele Fallin, Richard A Redett, Gerald Raymond, Holger Schwender, Sheng-Chih Jin, Margaret E Cooper, Martine Dunnwald, Maria A Mansilla, Elizabeth Leslie, Stephen Bullard, Andrew C Lidral, Lina M Moreno, Renato Menezes, Alexandre R Vieira, Aline Petrin, Allen J Wilcox, Rolv T Lie, Ethylin W Jabs, Yah Huei Wu-Chou, Philip K Chen, Hong Wang, Xiaoqian Ye, Shangzhi Huang, Vincent Yeow, Samuel S Chong, Sun Ha Jee, Bing Shi, Kaare Christensen, Mads Melbye, Kimberly F Doheny, Elizabeth W Pugh, Hua Ling, Eduardo E Castilla, Andrew E Czeizel, Lian Ma, L. Leigh Field, Lawrence Brody, Faith Pangilinan, James L Mills, Anne M Molloy, Peadar N Kirke, John M Scott, James M Scott, Mauricio Arcos-Burgos, and Alan F Scott. A genome-wide association study of cleft lip with and without cleft palate identifies risk variants near MAFB and ABCA4. Nat Genet, 42(6):525–529, Jun 2010.
- [2] Robert B. Scharpf, Terri H. Beaty, Holger Schwender, Samuel G. Younkin, Alan F. Scott, and Ingo Ruczinski. Fast detection of de novo copy number variants from SNP arrays for case-parent trios. *BMC Bioinformatics*, 13(1):330, Dec 2012.
- [3] Robert B Scharpf, Giovanni Parmigiani, Jonathan Pevsner, and Ingo Ruczinski. Hidden Markov models for the assessment of chromosomal alterations using high-throughput SNP arrays. *Annals of Applied Statistics*, 2(2):687–713, 2008.