# CCREPE: Compositionality Corrected by Permutation and Renormalization

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# **1** Introduction

*ccrepe* is a package for analysis of sparse compositional data. Specifically, it determines the significance of association between features in a composition, using any similarity measure (e.g. Pearson correlation, Spearman correlation, etc.) The CCREPE methodology stands for Compositionality Corrected by Renormalization and Permutation, as detailed below. The package also provides a novel similarity measure, the N-dimensional checkerboard score (NC-score), particularly appropriate to compositions derived from microbial community sequencing data. This results in p-values and false discovery rate q-values corrected for the effects of compositionality. The package contains two functions ccrepe and nc.score and is maintained by the Huttenhower lab (ccrepe-users@googlegroups.com).

## 2 ccrepe

ccrepe is the main package function. It calculates compositionality-corrected p-values and q-values for a user-selected similarity measure, operating on either one or two input matrices. If given one matrix, all features (columns) in the matrix are compared to each other using the selected similarity measure. If given two matrices, each feature in the first are compared against all features in the second.

#### 2.1 General functionality

Compositional data induces spurious correlations between features due to the nonindependence of values that must sum to a fixed total. CCREPE abrogates this when determining the significance of a similarity measure for each feature pair using two main steps, permutation/renormalization and bootstrapping. First, given two features to compare, CCREPE generates a null distribution of the similarity expected just due to compositionality by iteratively permuting one feature, renormalizing all samples in the composition to their previous sum, and computing the resulting similarity measures. Second, CCREPE bootstraps over sample subsets in order to assess confidence in the "true" similarity measure. Finally, the two resulting distributions are compared using a pooled-variance Z-test to give a compositionality-corrected p-value. False discovery rate q-values are additionally calculated using the Benjamin-Hochberg-Yekutieli procedure. For greater detail, see Faust et al. [2012] and Schwager and Colleagues.

#### 2.2 Arguments

- x First *dataframe* or *matrix* containing relative abundances. Columns are features, rows are samples. Rows should therefore sum to a constant. Row names are used for identification if present.
- y Second *dataframe* or *matrix* (optional) containing relative abundances. Columns are features, rows are samples. Rows should therefore sum to a constant. If both x and y are specified, they will be merged by row names. If no row names are specified for either or both datasets, the default is to merge by row number.
- sim.score Similarity measure, such as cor or nc.score. This can be either an existing R function or user-defined. If the latter, certain properties should be satisfied as detailed below (also see examples). The default similarity measure is Spearman correlation.
  - A user-defined similarity measure should mimic the interface of cor:
    - 1. Take either two vector inputs one matrix or dataframe input.
    - 2. In the case of two inputs, return a single number.
    - 3. In the case of one input, return a matrix in which the (i,j)th entry is the similarity score for column i and column j in the original matrix.
    - 4. The resulting matrix (in the case of one input) must be symmetric.
    - 5. The inputs must be named x and y.
- sim.score.args An optional list of arguments for the measurement function. When given, they are passed to the sim.score function directly. For example, in the case of cor, the following would be acceptable:

sim.score.args = list(method = "spearman", use = "complete.obs")

Note that this example corresponds to the default behavior.

- min.subj Minimum number (count) of samples that must be nonzero in order to apply the similarity measure. This is to ensure that there are sufficient samples to perform a bootstrap (default: 20).
- iterations The number of iterations for both bootstrap and permutation calculations (default: 1000).

subset.cols.x Subset of columns from x to work on. The default (NULL) uses all columns. Note that all features are used for normalization, but calculations are performed only with the requested subset.

- subset.cols.y Subset of columns from y to work on. The default (NULL) uses all columns. Note that all features are used for normalization, but calculations are performed only with the requested subset.
- errthresh1 Maximum allowable probability of getting all zeros in a given bootstrapped column for the first dataset x. If a feature has a number of zeros that makes the probability of obtaining all zeros when sampling with replacement greater than this value, that feature will be excluded from the subsequent analysis. This is to ensure that the standard deviation of the bootstrap sample is non-zero. (default: 0.0001).
- verbose If TRUE, print periodic progress of the algorithm through the dataset(s), as well as including more detailed debugging output. (default: FALSE).
- iterations.gap If verbose=TRUE, the number of iterations between issuing status messages (default: 100).
- distributions Optional output file for detailed log (if given) of all intermediate permutation and renormalization distributions.
- compare.within.x A boolean value indicating whether to do comparisons given by taking all subsets of size 2 from subset.cols.x or to do comparisons given by taking all possible combinations of subset.cols.x or subset.cols.y. If TRUE but subset.cols.y=NA, returns all comparisons involving any features in subset.cols.x.

concurrent.output Optional output file to which each comparison will be written as it is calculated.

make.output.table A boolean value indicating whether to include table-formatted output.

#### 2.3 Output

ccrepe returns a list containing both the calculation results and the parameters used:

- sim.score matrix of similarity scores for all requested comparisons. The (i,j)th element corresponds to the similarity
  score of column i (or the ith column of subset.cols.1) and column j (or the jth column of subset.cols.1)
  in one dataset, or to the similarity score of column i (or the ith column of subset.cols.1) in dataset x and
  column j (or the jth column of subset.cols.2) in dataset y in the case of two datasets.
- p.values *matrix* of the corrected p-values for all requested comparisons. The (i,j)th element corresponds to the p-value of the (i,j)th element of sim.score.
- q.values matrix of the Benjamini-Hochberg-Yekutieli corrected p-values. The (i,j)th element corresponds to the p-value of the (i,j)th element of sim.score.

#### 2.4 Usage

ccrepe(x = NA, y = NA, sim.score = cor, sim.score.args = list(), min.subj = 20, iterations = 1000, subset.cols.x = NULL, subset.cols.y = NULL, errthresh1 = 1e-04, verbose = FALSE, iterations.gap = 100, distributions = NA, compare.within.x = TRUE, concurrent.output = NA, make.output.table = FALSE)

#### 2.5 Example 1

An example of how to use ccrepe with one dataset.

```
data <- matrix(rlnorm(40, meanlog = 0, sdlog = 1), nrow = 10, ncol = 4)
data[, 1] = 2 * data[, 2] + rnorm(10, 0, 1)
data.rowsum <- apply(data, 1, sum)
data.norm <- data/data.rowsum</pre>
```

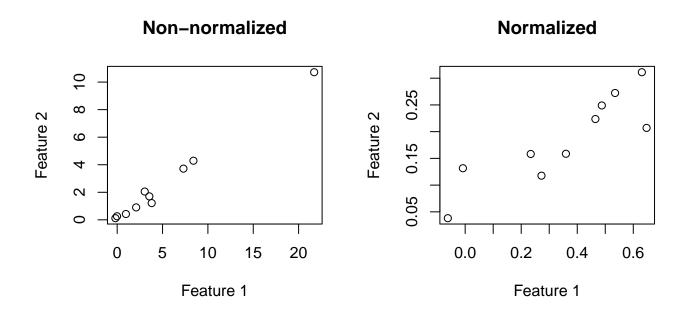


Figure 1: Non-normalized and normalized associations between feature 1 and feature 2. In this case we would expect feature 1 and feature 2 to be associated. In the output we see this by the positive sim.score value in the [1,2] element of test.output\$sim.score and the small q-value in the [1,2] element of test.output\$g.values.

```
apply(data.norm, 1, sum) # The rows sum to 1, so the data are normalized
## [1] 1 1 1 1 1 1 1 1 1 1
test.input <- data.norm</pre>
dimnames(test.input) <- list(c("Sample 1", "Sample 2", "Sample 3", "Sample 4",
    "Sample 5", "Sample 6", "Sample 7", "Sample 8", "Sample 9", "Sample 10"), c("Feature 1",
    "Feature 2", "Feature 3", "Feature 4"))
test.output <- ccrepe(x = test.input, iterations = 20, min.subj = 10)</pre>
par(mfrow = c(1, 2))
plot(data[, 1], data[, 2], xlab = "Feature 1", ylab = "Feature 2", main = "Non-normalized")
plot(data.norm[, 1], data.norm[, 2], xlab = "Feature 1", ylab = "Feature 2", main = "Normalized")
test.output
## $p.values
             Feature 1 Feature 2 Feature 3 Feature 4
##
## Feature 1
                   NA 0.0561932 0.0002923 0.004755
## Feature 2 0.0561932
                              NA 0.0009184 0.023510
## Feature 3 0.0002923 0.0009184
                                        NA 0.200533
## Feature 4 0.0047548 0.0235104 0.2005330
                                                   NA
##
## $z.stat
##
             Feature 1 Feature 2 Feature 3 Feature 4
## Feature 1
                    NA
                           1.910
                                    -3.622
                                               -2.823
                                    -3.314
                                              -2.265
## Feature 2 1.910
                          NA
```

```
## Feature 3 -3.622 -3.314 NA
                                      1.280
## Feature 4 -2.823
                    -2.265
                              1.280
                                        NA
##
## $q.values
## Feature 1 Feature 2 Feature 3 Feature 4
## Feature 1 NA 0.159744 0.004155 0.02253
## Feature 2 0.159744 NA 0.006527 0.08354
## Feature 3 0.004155 0.006527 NA 0.47506
## Feature 4 0.022528 0.083543 0.475058
                                      NA
##
## $sim.score
## Feature 1 Feature 2 Feature 3 Feature 4
## Feature 1 NA 0.8424 -0.8909 -0.8424
## Feature 2 0.8424 NA -0.8424 -0.7818
## Feature 3 -0.8909 -0.8424 NA
                                   0.6000
## Feature 4 -0.8424 -0.7818 0.6000 NA
```

#### 2.6 Example 2

An example of how to use ccrepe with two datasets.

```
data <- matrix(rlnorm(40, meanlog = 0, sdlog = 1), nrow = 10, ncol = 4)</pre>
data[, 1] = 2 * data[, 2] + rnorm(10, 0, 1)
data.rowsum <- apply(data, 1, sum)</pre>
data.norm <- data/data.rowsum</pre>
apply(data.norm, 1, sum) # The rows sum to 1, so the data are normalized
## [1] 1 1 1 1 1 1 1 1 1 1
test.input <- data.norm</pre>
data2 <- matrix(rlnorm(105, meanlog = 0, sdlog = 1), nrow = 15, ncol = 7)</pre>
aligned.rows <- c(seq(1, 4), seq(6, 9), 11, 12) # The datasets dont need
# to have subjects line up exactly
data2[aligned.rows, 1] <- 2 * data[, 3] + rnorm(10, 0, 1)
data2.rowsum <- apply(data2, 1, sum)</pre>
data2.norm <- data2/data2.rowsum</pre>
apply(data2.norm, 1, sum) # The rows sum to 1, so the data are normalized
## [1] 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
test.input.2 <- data2.norm</pre>
dimnames(test.input) <- list(paste("Sample", seq(1, 10)), paste("Feature", seq(1,</pre>
    4)))
dimnames(test.input.2) <- list(paste("Sample", c(seq(1, 4), 11, seq(5, 8), 12,</pre>
    9, 10, 13, 14, 15)), paste("Feature", seq(1, 7)))
test.output.two.datasets <- ccrepe(x = test.input, y = test.input.2, iterations = 20,</pre>
  \min.subj = 10)
par(mfrow = c(1, 2))
plot(data2[aligned.rows, 1], data[, 3], xlab = "dataset 2: Feature 1", ylab = "dataset 1: Feature 3",
main = "Non-normalized")
```

```
5
```

```
plot(data2.norm[aligned.rows, 1], data.norm[, 3], xlab = "dataset 2: Feature 1",
    ylab = "dataset 1: Feature 3", main = "Normalized")
```

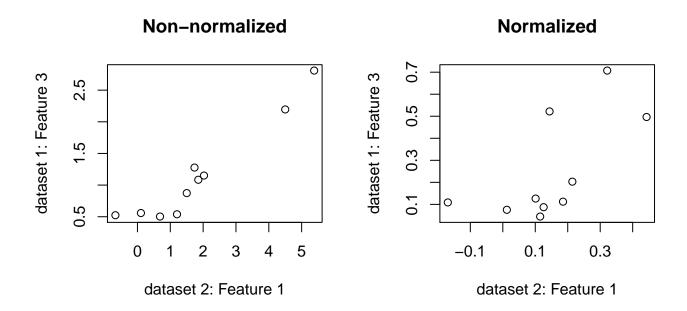


Figure 2: Non-normalized and normalized associations between feature 1 and feature 2. In this case we would expect feature 1 and feature 2 to be associated. In the output we see this by the positive sim.score value in the [1,2] element of test.output\$sim.score and the small q-value in the [1,2] element of test.output\$q.values.

```
test.output.two.datasets
## $p.values
##
             Feature 1 Feature 2 Feature 3 Feature 4 Feature 5 Feature 6
                                                         0.04956
## Feature 1
             0.104140
                           0.9989
                                     0.7591
                                               0.85428
                                                                     0.3550
##
  Feature 2
              0.683020
                           0.8943
                                     0.4555
                                               0.21462
                                                         0.39688
                                                                     0.6196
## Feature 3
              0.006579
                           0.2048
                                     0.8568
                                               0.09158
                                                         0.19884
                                                                     0.6785
##
  Feature 4
              0.087169
                           0.8504
                                     0.6878
                                               0.03181
                                                         0.09084
                                                                     0.6666
##
             Feature 7
## Feature 1
               0.77123
## Feature 2
               0.45718
## Feature 3
               0.28698
## Feature 4
               0.01626
##
## $z.stat
##
             Feature 1 Feature 2 Feature 3 Feature 4 Feature 5 Feature 6
## Feature 1
               -1.6251 -0.001415
                                     0.3067
                                               -0.1837
                                                          1.9637
                                                                     0.9250
                                                                    -0.4964
## Feature 2
                0.4083 -0.132818
                                     0.7463
                                                1.2410
                                                         -0.8472
## Feature 3
                2.7175 -1.268118
                                    -0.1804
                                               1.6871
                                                         -1.2849
                                                                    -0.4144
               -1.7105 -0.188625
## Feature 4
                                     0.4018
                                               -2.1468
                                                          1.6910
                                                                     0.4308
##
             Feature 7
## Feature 1
                0.2908
               -0.7435
## Feature 2
## Feature 3
               -1.0648
## Feature 4
              2.4031
```

```
##
## $q.values
## Feature 1 Feature 2 Feature 3 Feature 4 Feature 5 Feature 6
## Feature 1 1.4249 3.905 3.777 3.741 1.356 2.989
## Feature 2 3.7383 3.626 3.324 2.136 3.103 3.990
## Feature 30.72012.2413.6071.4322.418## Feature 41.90843.8793.5851.1611.657
                                              2.418
                                                       3.909
                                                     4.054
##
   Feature 7
## Feature 1 3.6705
## Feature 2 3.1278
## Feature 3 2.6178
## Feature 4 0.8897
##
## $sim.score
## Feature 1 Feature 2 Feature 3 Feature 4 Feature 5 Feature 6
## Feature 1 -0.4545 0.09091 0.1394 -0.2485 0.4909 0.23636
## Feature 2 0.1030 0.17576 0.2485 0.3576 -0.3576 0.12727
## Feature 3 0.6970 -0.64848 -0.1758 0.5636 -0.3818 -0.11515
## Feature 4 -0.5030 -0.04242 0.2727 -0.5879 0.4909 0.01818
## Feature 7
## Feature 1 0.1515
## Feature 2 -0.4667
## Feature 3 -0.0303
## Feature 4 0.5394
```

#### 2.7 Example 3

An example of how to use ccrepe with nc.score as the similarity score.

```
data <- matrix(rlnorm(40, meanlog = 0, sdlog = 1), nrow = 10, ncol = 4)</pre>
data[, 1] = 2 * data[, 2] + rnorm(10, 0, 1)
data.rowsum <- apply(data, 1, sum)</pre>
data.norm <- data/data.rowsum</pre>
apply(data.norm, 1, sum) # The rows sum to 1, so the data are normalized
## [1] 1 1 1 1 1 1 1 1 1 1
test.input <- data.norm</pre>
dimnames(test.input) <- list(paste("Sample", seq(1, 10)), paste("Feature", seq(1,
    4)))
test.output.nc.score <- ccrepe(x = test.input, sim.score = nc.score, iterations = 20,</pre>
 min.subj = 10)
par(mfrow = c(1, 2))
plot(data[, 1], data[, 2], xlab = "Feature 1", ylab = "Feature 2", main = "Non-normalized")
plot(data.norm[, 1], data.norm[, 2], xlab = "Feature 1", ylab = "Feature 2", main = "Normalized")
test.output.nc.score
## $p.values
           Feature 1 Feature 2 Feature 3 Feature 4
##
               NA 0.01564 0.03470 0.0231
## Feature 1
## Feature 2 0.01564 NA 0.05536 0.2074
```

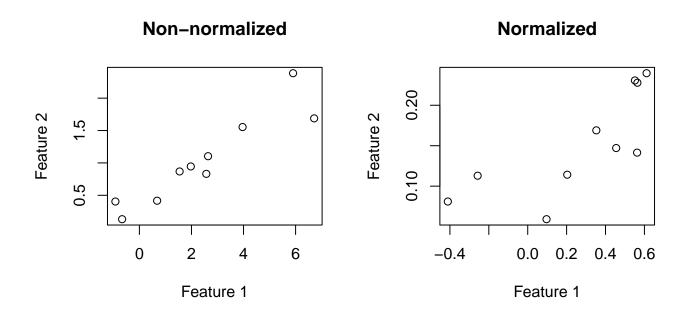


Figure 3: Non-normalized and normalized associations between feature 1 and feature 2. In this case we would expect feature 1 and feature 2 to be associated. In the output we see this by the positive sim.score value in the [1,2] element of test.output\$sim.score and the small q-value in the [1,2] element of test.output\$q.values. In this case, however, the sim.score represents the NC-Score between two features rather than the Spearman correlation.

##	Feature	3	0.03470	0.05536	NA	0.4662	
##	Feature	4	0.02310	0.20737	0.46624	NA	
##							
##	\$z.stat						
##			Feature 1	Feature 2	Feature 3	Feature 4	
##	Feature	1	NA	2.417	-2.1119	-2.2718	
##	Feature	2	2.417	NA	-1.9160	-1.2608	
##	Feature	3	-2.112	-1.916	NA	0.7286	
##	Feature	4	-2.272	-1.261	0.7286	NA	
##							
##	\$q.values						
##			Feature 1	Feature 2	Feature 3	Feature 4	
##	Feature	1	NA	0.2224	0.1644	0.1642	
##	Feature	2	0.2224	NA	0.1967	0.5895	
##	Feature	3	0.1644	0.1967	NA	1.1045	
##	Feature	4	0.1642	0.5895	1.1045	NA	
##							
##	\$sim.score						
##			Feature 1	Feature 2	Feature 3	Feature 4	
##	Feature	1	NA	0.7792	-0.7489	-0.5887	
##	Feature	2	0.7792	NA	-0.8355	-0.2857	
##	Feature	3	-0.7489	-0.8355	NA	0.1732	
##	Feature	4	-0.5887	-0.2857	0.1732	NA	

#### 2.8 Example 4

An example of how to use ccrepe with a user-defined sim.score function.

```
data <- matrix(rlnorm(40, meanlog = 0, sdlog = 1), nrow = 10, ncol = 4)</pre>
data[, 1] = 2 * data[, 2] + rnorm(10, 0, 1)
data.rowsum <- apply(data, 1, sum)</pre>
data.norm <- data/data.rowsum</pre>
apply(data.norm, 1, sum) # The rows sum to 1, so the data are normalized
## [1] 1 1 1 1 1 1 1 1 1 1
test.input <- data.norm</pre>
dimnames(test.input) <- list(paste("Sample", seq(1, 10)), paste("Feature", seq(1,</pre>
   4)))
my.test.sim.score <- function(x, y = NA, constant = 0.5) {</pre>
   if (is.vector(x) && is.vector(y))
       return(constant)
   if (is.matrix(x) && is.na(y))
       return(matrix(rep(constant, ncol(x)^2), ncol = ncol(x)))
   if (is.data.frame(x) && is.na(y))
       return(matrix(rep(constant, ncol(x)^2), ncol = ncol(x))) else stop("ERROR")
}
test.output.sim.score <- ccrepe(x = test.input, sim.score = my.test.sim.score,</pre>
iterations = 20, min.subj = 10, sim.score.args = list(constant = 0.6))
par(mfrow = c(1, 2))
plot(data[, 1], data[, 2], xlab = "Feature 1", ylab = "Feature 2", main = "Non-normalized")
plot(data.norm[, 1], data.norm[, 2], xlab = "Feature 1", ylab = "Feature 2", main = "Normalized")
test.output.sim.score
## $p.values
## Feature 1 Feature 2 Feature 3 Feature 4
## Feature 1 NA NaN NaN NaN
                         NA
## Feature 2
                NaN
                                  NaN
                                           NaN
                         NaN
## Feature 3
                NaN
                                            NaN
                                   NA
## Feature 4
                NaN
                         NaN
                                   NaN
                                            NA
##
## $z.stat
## Feature 1 Feature 2 Feature 3 Feature 4
## Feature 1 NA NaN NaN NaN
## Feature 2
               NaN
                          NA
                                   NaN
                                            NaN
                         NaN
               NaN
## Feature 3
                                   NA
                                             NaN
## Feature 4
                NaN
                         NaN
                                    NaN
                                            NA
##
## $q.values
##
    Feature 1 Feature 2 Feature 3 Feature 4
## Feature 1 NA NaN NaN
                                            NaN
## Feature 2
               NaN
                         NA
                                   NaN
                                             NaN
                        NaN
               NaN
                                   NA
## Feature 3
                                             NaN
## Feature 4
                NaN
                         NaN
                                    NaN
                                             NA
##
```

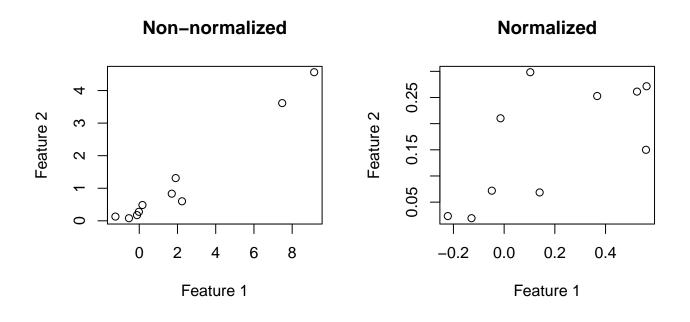


Figure 4: Non-normalized and normalized associations between feature 1 and feature 2. In this case we would expect feature 1 and feature 2 to be associated. Note that the values of sim.score are all 0.6 and none of the p-values are very small because of the arbitrary definition of the similarity score.

## \$sim.score							
##			Feature 1	Feature 2	Feature 3	Feature 4	
##	Feature	1	NA	0.6	0.6	0.6	
##	Feature	2	0.6	NA	0.6	0.6	
##	Feature	3	0.6	0.6	NA	0.6	
##	Feature	4	0.6	0.6	0.6	NA	

#### 2.9 Example 5

```
An example of how to use ccrepe when specifying column subsets.
data <- matrix(rlnorm(40, meanlog = 0, sdlog = 1), nrow = 10, ncol = 4)
data.rowsum <- apply(data, 1, sum)
data.norm <- data/data.rowsum
apply(data.norm, 1, sum) # The rows sum to 1, so the data are normalized
## [1] 1 1 1 1 1 1 1 1 1
test.input <- data.norm
dimnames(test.input) <- list(paste("Sample", seq(1, 10)), paste("Feature", seq(1,
4)))
test.output.1.3 <- ccrepe(x = test.input, iterations = 20, min.subj = 10, subset.cols.x = c(1,
3))
test.output.1 <- ccrepe(x = test.input, iterations = 20, min.subj = 10, subset.cols.x = c(1),
compare.within.x = FALSE)
test.output.12.3 <- ccrepe(x = test.input, iterations = 20, min.subj = 10, subset.cols.x = c(1),</pre>
```

```
2), subset.cols.y = c(3), compare.within.x = FALSE)
test.output.1.3$sim.score
##
            Feature 1 Feature 2 Feature 3 Feature 4
## Feature 1
              NA NA -0.006061
                                                 NA
                   NA
                                                 NA
## Feature 2
                             NΑ
                                       NΑ
## Feature 3 -0.006061
                             NA
                                       NA
                                                  NA
## Feature 4
                   ΝA
                             NA
                                       NΑ
                                                  NΑ
test.output.1$sim.score
##
            Feature 1 Feature 2 Feature 3 Feature 4
                                            -0.7212
## Feature 1
                   NA
                       -0.4182 -0.006061
## Feature 2 -0.418182
                             NA
                                       NA
                                                  NA
## Feature 3 -0.006061
                             NA
                                       NA
                                                  NA
## Feature 4 -0.721212
                             NA
                                       NΑ
                                                  NA
test.output.12.3$sim.score
##
            Feature 1 Feature 2 Feature 3 Feature 4
## Feature 1
                   NA
                             NA -0.006061
                                                  NA
## Feature 2
                   NA
                             NA -0.078788
                                                 NA
## Feature 3 -0.006061 -0.07879
                                                 NA
                                       NΑ
## Feature 4 NA
                             NA
                                       NA
                                                  NA
```

### 3 nc.score

The nc.score similarity measure is an N-dimensional extension of the checkerboard score particularly suited to similarity score calculations between compositions derived from ecological relative abundance measurements. In such cases, features typically represent species abundances, and the NC-score discretizes these continuous values into one of N bins before computing a normalized similarity of co-occurrence or co-exclusion. This can be used as a standalone function or with ccrepe as above to obtain compositionality-corrected p-values.

#### 3.1 General Functionality

The NC-score is an extension to Diamond's checkerboard score (see Cody and Diamond [1975]) to ordinal data. The generalization of the checkerboard score is through defining general co-variation and co-exclusion patterns in ordinal data. With ordinal data, the checkerboard considers the  $2 \times 2$  submatrices which are of the form:

$$\left[\begin{array}{cc}a&b\\c&d\end{array}\right]$$

In the cases that a < b and c < d (or a > b and c > d), the submatrix is considered a co-variation pattern; conversely, if a > b and c < d (or a < b and c > d) then the submatrix is considered a co-exclusion pattern. The number of co-exclusion patterns for n bins is adjusted based on the expected ratio of co-variation to co-exclusion patterns in any given sample. The adjustment factor is given by:

$$1.5 * \frac{n(n-1)}{n^2 - n + 1},$$

with the complete derivation being left to Bielski [2013].

The function as implemented here first performs basic quality control filtering of input relative abundance data. It then transforms relative abundances to ordinal values based on user-provided or default bin thresholds. It then computes a raw NC-score:

$$C_{ij} - 1.5 * \frac{n(n-1)}{n^2 - n + 1} * D_{ij}$$

where  $C_{ij}$  and  $D_{ij}$  are the total number of co-variation and co-exclusion patterns, respectively, between species i and j and where n is the number of bins.

The raw NC-score is then normalized to between -1 and 1 for n bins and s samples by dividing by the maximum possible NC-score, which is based on the maximum number of co-variation patterns possible. The normalized NC-score is then analogous to Pearsons  $\rho$ , where positive values indicate more co-variation than co-exclusion patterns and the magnitude of the score indicates the strength of the association between the species. The derivation of this normalization factor can be found in Bielski [2013]. The normalization factor is given by:

$$\begin{pmatrix} s \\ 2 \end{pmatrix} - \left[ s \mod n * \begin{pmatrix} \lfloor \frac{s}{n} \rfloor + 1 \\ 2 \end{pmatrix} + (n - s \mod n) * \begin{pmatrix} \lfloor \frac{s}{n} \rfloor \\ 2 \end{pmatrix} \right].$$

#### 3.2 Arguments

- x First numerical *vector*, or single *dataframe* or *matrix*, containing relative abundances. If the latter, columns are features, rows are samples. Rows should therefore sum to a constant.
- y If provided, second numerical vector containing relative abundances. If given, x must be a vector as well.
- bins Either the number of bins to use or a *vector* specifying bin edges. If a single number is given, this is used as the number of bins with the discretize function of the package *infotheo*. If a *vector* is specified, the function findInterval is used to discretize the data. The default behavior is to use the defaults for the discretize function.

verbose Request verbose output.

- min.abundance Minimum abundance threshold for quality control filtering. For a feature to be included, it must take a value of at least min.abundance in at least min.samples percent of samples.
- min.samples Minimum sample threshold for quality control filtering. For a feature to be included, it must take a value of at least min.abundance in at least min.samples percent of samples.

#### 3.3 Output

nc.score returns either a single number (if called with two vectors) or a *matrix* of all pairwise scores (if called with a *matrix*) of normalized scores.

#### 3.4 Usage

nc.score(x = NA, y = NA, bins = NA, verbose = FALSE, min.abundance = 1e-04, min.samples = 0.1)

#### 3.5 Example 1

An example of using nc.score to get a single similarity score or a matrix.

```
data <- matrix(rlnorm(40, meanlog = 0, sdlog = 1), nrow = 10, ncol = 4)
data.rowsum <- apply(data, 1, sum)
data[, 1] = 2 * data[, 2] + rnorm(10, 0, 1)
data.norm <- data/data.rowsum</pre>
```

```
apply(data.norm, 1, sum) # The rows sum to 1, so the data are normalized
## [1] 0.7375 1.9116 1.2889 0.6011 1.3995 0.9039 1.1886 1.1191 2.3321 1.0530
test.input <- data.norm
dimnames(test.input) <- list(paste("Sample", seq(1, 10)), paste("Feature", seq(1,
4)))
test.output.matrix <- nc.score(x = test.input)
test.output.num <- nc.score(x = test.input[, 1], y = test.input[, 2])
par(mfrow = c(1, 2))
plot(data[, 1], data[, 2], xlab = "Feature 1", ylab = "Feature 2", main = "Non-normalized")
plot(data.norm[, 1], data.norm[, 2], xlab = "Feature 1", ylab = "Feature 2", main = "Normalized")
```

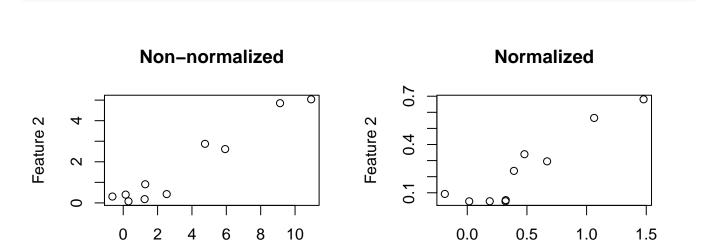


Figure 5: Non-normalized and normalized associations between feature 1 and feature 2 of the second example. Again, we expect to observe a positive association between feature 1 and feature 2. In terms of generalized checkerboard scores, we would expect to see more co-variation patterns than co-exclusion patterns. This is shown by the positive and relatively high value of the [1,2] element of test.output.matrix (which is identical to test.output.num)

Feature 1

test.output.matrix ## Feature 1 Feature 2 Feature 3 Feature 4 0.77922 -0.33333 -0.3333 ## Feature 1 NA ## Feature 2 0.7792 NA -0.09524 -0.2857 -0.3333 -0.3333 ## Feature 3 -0.09524 NA **##** Feature 4 -0.3333 -0.28571 -0.33333 NA test.output.num ## [1] 0.7792

Feature 1

#### 3.6 Example 2

An example of using nc.score with an aribitrary bin number.

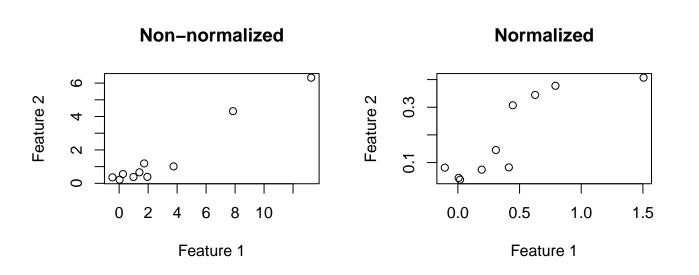


Figure 6: Non-normalized and normalized associations between feature 1 and feature 2 of the second example. Again, we expect to observe a positive association between feature 1 and feature 2. In terms of generalized checkerboard scores, we would expect to see more co-variation patterns than co-exclusion patterns. This is shown by the positive and relatively high value in the [1,2] element of test.output. In this case, the smaller bin number yields a smaller NC-score because of the coarser partitioning of the data.

test.output							
##		Feature 1	Feature 2	Feature 3	Feature 4		
## Feature	1	NA	0.6	-0.2	-0.2		
## Feature	2	0.6	NA	0.2	-0.2		
## Feature	3	-0.2	0.2	NA	0.6		
## Feature	4	-0.2	-0.2	0.6	NA		

#### 3.7 Example 3

An example of using nc.score with user-defined bin edges.

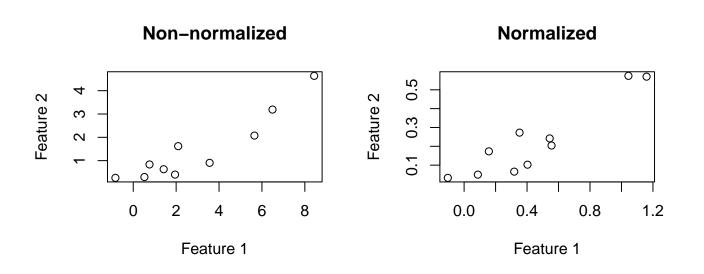


Figure 7: Non-normalized and normalized associations between feature 1 and feature 2 of the second example. Again, we expect to observe a positive association between feature 1 and feature 2. In terms of generalized checkerboard scores, we would expect to see more co-variation patterns than co-exclusion patterns. This is shown by the positive and relatively high value in the [1,2] element of test.output. The bin edges specified here represent almost absent ([ 0,0.001)), low abundance ([0.001,0.1)), medium abundance ([0.1,0.25)), and high abundance ([0.6,1)).

test.output							
##		Feature 1	Feature 2	Feature 3	Feature 4		
## Feature	1	NA	0.6250	-0.5107	-0.02857		
## Feature	2	0.62500	NA	-0.2017	-0.44589		
## Feature	3	-0.51071	-0.2017	NA	0.12682		
## Feature	4	-0.02857	-0.4459	0.1268	NA		

## 4 References

# References

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